Progressive Image Compression With Bandelets

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Abstract. Image compression has emerged as a major research area due to the phenomenal growth of applications that generate, process and transmit images. Image compression can be sequential or progressive. Progressive compression techniques generate an embedded bit stream and the fidelity of the reconstruction depends on the number of bits received and decoded. Natural images contain edges, geometry, texture and other discontinuities / details that are oriented in various directions. The state-of-the-art wavelet transform captures point singularities, but not along surfaces with geometric regularity. The second generation discrete wavelet-bandelet transform is proposed to overcome the drawback of wavelets in higher dimensions and capture the geometry in images. The redundancy in the wavelet transform is removed by bandeletization. The wavelet-bandelet coefficients are quantized and encoded using modified bit plane coding and the results have been compared with the existing bit plane coding and the set partitioning in hierarchical trees algorithm. Bandelets produce superior visual quality in the reconstructed image than wavelets. The parameters used for the evaluation of the algorithm are compression ratio, bits per pixel and peak signal-to-noise ratio.

Keywords: Image Compression, Progressive, Wavelets, Bandelets, Bit Plane Coding, SPIHT.

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1 Introduction

Bandelet transform is an orthogonal, multi-scale transform that captures the geometry in images. When there are edges in the image, there are sharp transitions across edges but along the edges there are regular variations, called geometric flow. The geometry of natural surfaces can be modeled as a function that is C^2 regular (C is a constant) outside a set of edges that are also regular. In Figure 1 (a) - (d) shows some examples of images that are geometrically regular and also contain texture. Taking advantage of geometrical structures in natural images improves the efficiency of the compression, since the human perception is sensitive to curves and other geometric features.

Wavelet transform compresses the regular parts of an image well but produces coefficients with high values near singularities. Fig. 2(a) - (c) shows the difference between wavelet approximation and a triangular basis function in capturing an image which is geometrically regular. The bandelet transform applied directly on the image at a fixed scale is the first generation mono-resolution discrete bandelet transform (DBT). The drawback of the DBT is that it is applied on blocks of the image and produces blocking artifacts at low bitrates. The second generation discrete wavelet-bandelet transform (DWBT) is proposed to overcome the drawback of mono-resolution bandelets. The orthogonal wavelet filter bank, followed by adaptive, geometric, orthogonal filters is used in obtaining wavelet-bandelet transform. The second generation bandelets are simpler, orthogonal and do not produce blocking artifacts. The redun-

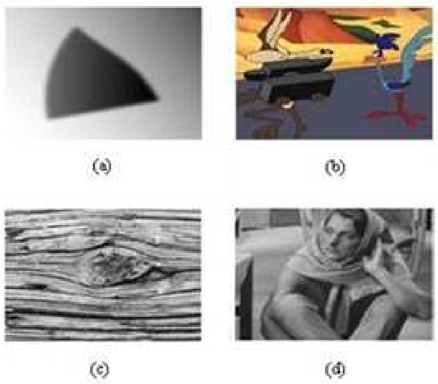


Figure 1: Images with geometry and texture (a) Geometrically regular image, (b) Cartoon, (c) Wood texture, (d) Barbara.

dancy in the discrete wavelet transform (DWT) is removed by bandeletization.

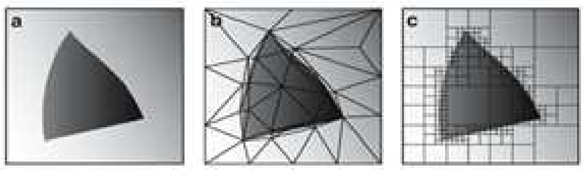


Figure 2: (a) Geometrically regular image (b) Triangular approximation (c) Wavelet approximation

In the existing literature [2], Le Pennec and Mallat have developed a double layer algorithm using wavelets and bandelets, and presented with examples. In [3] the same authors have produced a sparse image representation using the geometric regularity of images and compared the performance of wavelet and bandelet coders. Wavelets introduce visible ringing effects, whereas bandelets do not. In another paper [4], Le Pennec and Mallat have done a rigorous analysis of bandelet bases and presented mathematical proof that the bandelet approximation satisfies an optimal asymptotic error decay rate. They have used quantized bandelet coefficients for coding edges and wavelets for coding smooth regions of the image. Peyre and Mallat have published several papers [6][7][8][9][10] on bandelets with basic concepts and rigorous mathematical analysis of the bandelet transform and its application to image compression and denoising. They have demonstrated the superiority of bandelets over wavelets in the visual quality of reconstructed images. They have also given an insight into the MATLAB implementation of bandelet transform and the listing of the source code that is available in MATLAB Central. In [5] Liu et al have applied bandelets and SPIHT for Synthetic Aperture Radar (SAR) image compression using a multi-layered image representation.

In this work, it is proposed to compress the coefficients of the DWBT using modified bit plane coding (MBPC) and the results have been compared with the existing bit plane coding (BPC) specified in JPEG2000 given in [13] and also with SPIHT as in [11]. It is found that the modified algorithm is superior in the reconstructed image quality compared to the other two algorithms. In this paper, the proposed algorithm has been evaluated using images for which this method is suitable. Section 2 gives an overview of the non-linear bandelets and the computation of the discrete wavelet-bandelet transform. Section 3 presents the proposed algorithm and Section 4 compares the results of the proposed algorithm with the existing algorithms. The paper concludes with Section 5.

2 Computation of Wavelet-Bandelet Transform

Bandelet bases are elongated in the direction of geometric flow, adapted for image geometry, with compact support. The human perception is sensitive to curves and other geometric features in the image and this is utilized in compression with bandelets. The geometry must be estimated from discrete image samples. The bandelet decomposition is computed with a geometric orthogonal transform that has basis functions elongated along the singularity, whereas the orthogonal wavelet transform has basis functions with a square support. The second generation bandelets apply bandeletization on subbands of the wavelet decomposition and removes the redundancy in the wavelet coefficients. The subbands of the wavelet decomposition are divided into dyadic squares. This is done by searching for a regularity flow in a direction along which the function is as regular as possible, as shown in Fig. 3.

To represent the image partition with few parameters and be able to compute an optimal partition with a fast algorithm, the image is partitioned into squares of varying dyadic sizes. The dyadic squares image segmentation is obtained by successive quadtree segmentation of square regions into four squares of half width on each wavelet scale. A square subdivided into four smaller squares corresponds to a node having four children in the quadtree. Fig. 4 gives an example of dyadic square image segmentation with the corresponding

quadtree. The best geometry or direction is selected in each square by exhaustively searching all possible directions and computing the Lagrangian for every direc-

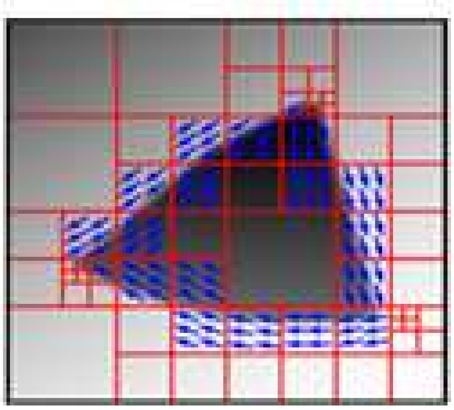


Figure 3: Geometric Flow in Dyadic Squares

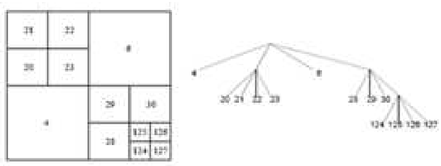


Figure 4: Quadtree Segmentation

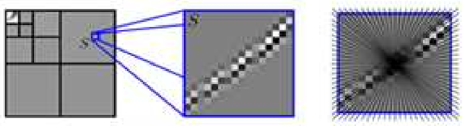


Figure 5: (a) 2D DWT (b) Coefficients within square S along geometry (c) Possible directions within S

tion. The Lagrangian optimization function is defined with the approximation error and bit-rate required for encoding. The direction that minimizes the Lagrangian function produces the least approximation error and is the best direction for the given bit-rate. The bandelet transform is applied on each leaf of the quadtree in the specified best direction to produce the bandelet coefficients. This is the DWBT that removes the geometric redundancy of orthogonal wavelet coefficients. The bandelet transform coding scheme has an error decay rate that is asymptotically optimal for geometrically regular images. The 1D Haar transform applied on the wavelet coefficients in a specified direction is the warped Haar transform. This is the bandeletization that removes the correlation between wavelet coefficients near singularities (anisotropic redundancy). Fig. 5(a) shows the subband structure resulting from the 2D DWT applied on the input image. Fig. 5(b) shows the coefficients within the square S along the direction of geometry (directional projection). The possible directions within the square S are shown in Fig. 5(c).

Each sampling location of the regular grid is projected orthogonal to the geometry direction to get a new point (Fig. 6(a)). The new points are ordered according to numbering along the perpendicular axis. The new 1D signal is shown in Fig. 6(b). The 1D wavelet transform is applied on the projected samples and the resulting coefficients are thresholded to produce a sparse representation, as shown in Fig. 7(a) and (b). The threshold determines the compression rate of the algorithm. The bandelet coefficients are quantized and encoded along with the dyadic segmentation and polynomial flow information. For a fixed quantization step size, the best basis that minimizes the distortion-rate is found using Lagrangian optimization. The number of bits for coding is the sum of the bits for representing the position and width of the square, bits for geometric flow and bits for the quantized bandelet coefficients. The segmentation of the image support $R_{S,}$ geometric flow in each region of the support R_G and the quantized bandelet coefficients R_B are all encoded and contribute to the total bit-rate R. The number of bits R_S is equal to the number of nodes in the quadtree. R_G is the sum of the bits in all the squares where a flow is defined.

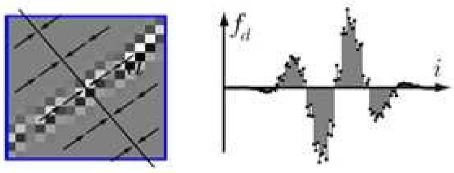


Figure 6: (a) Projection of coefficients (b) Resulting 1D signal of projected coefficients

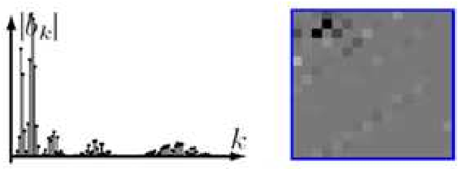


Figure 7: (a) 1D warped Haar transform coefficients (b) Bandelet coefficients

Similar to the compression in wavelet bases, it is required to quantize uniformly the bandelet coefficients with a quantization step T. The reconstructed image f_R is given by,

$$f_R = \sum_v Q_T(\langle f, b_v \rangle) b_v \quad (1)$$

where f is the original image, b_v is the bandelet basis and R is the number of bits needed to specify f_R. Q_T is a uniform quantizer defined by,

$$Q_T(x) = qT, if \left(q - \frac{1}{2}\right) T \leqslant x \leqslant \left(q + \frac{1}{2}\right) T \quad (2)$$

The distortion of this coding scheme is $D_b(R) = \|f - f_R\|^2$ and for a given bit budget R it is necessary to find the best bandelet basis that gives the lowest distortion.

3 Proposed Algorithm

This section presents the algorithm proposed in this work for progressive image compression with bandelets. It includes the following key steps:

(a) Discrete Wavelet Transform
(b) Bandeletization
(c) Quantization (with modified step size)
(d) Bit Plane Coding

The input image is transformed using the DWT for three levels of decomposition. The bandelet transform is applied on each subband, producing a sparser representation. This is the second generation wavelet-bandelets that acquires the texture information / geometry present in the image. Texture or geometry refers to patterns in the image that are oriented at various directions. As there are three levels of decomposition involved, three step sizes are allotted to the quantizer, one for each level. The quantizer step sizes based on the JPEG2000 standard is given below:

$$q(i,j) = sign[p(i,j)] \lfloor \frac{|p(i,j)|}{\Delta_b} + e \rfloor \quad (3)$$

$$\Delta_b = bss \times \sqrt{\frac{1}{G_b}}, G_b = 2^{2 \times level} \quad (4)$$

where $p(i,j)$ is the input coefficient, $q(i,j)$ is the quantized coefficient, bss is the base step size, Δ_b is the step size for subband b, e is a constant and $level$ is the level of decomposition.

The third level, which contains the maximum (most important) information in the image, is allotted the smallest step size and it is progressively increased at the lower levels. This quantization makes the compression lossy, but the computational complexity of the coding process is significantly reduced and the compression achieved is more. In the modified algorithm, the step size is chosen as constant for the subbands at all the levels. This improves the quality of the reconstructed image with DWBT as compared to the JPEG2000 standard. The parameters [1][12] used in the evaluation of the algorithm are: compression ratio (CR), bits per pixel (bpp) and peak signal-to-noise ratio (PSNR). CR is defined as the ratio of number of bits representing the image without compression to the number of bits representing the image with compression. bpp is defined as the average number of bits used to represent each pixel in the image. PSNR is defined as the ratio of the square of the peak value of the image to the mean squared error, expressed in decibels (dB). Mean squared error is defined as the average value of the square of the difference between the original pixel values and the reconstruction values.

The original images that have been used for performance evaluation are given in Fig. 8(a) and (b). The results of encoding by applying DWT and DWBT with BPC specified in JPEG2000 standard and the proposed algorithm is compared in Table 1 and Table 2 for the images *Barbara* and *Ridge on Eros* respectively.

Figure 8: Original Test Images: (a) Barbara (b) Ridge on EROS

The reconstructed images for the above compres-

Table 1: Comparison of the proposed algorithm results with DWT / BPC, DWBT / BPC, *Barbara*, PSNR(dB)

bpp	$DWT/$ BPC	$DWBT/$ BPC	$DWBT/$ $MBPC$
0.1	23.53	23.84	24.33
0.2	25.31	25.68	26.46
0.3	26.38	27.11	28.53
0.4	29.16	28.62	30.19
0.5	30.52	30.33	31.44
0.6	31.54	31.36	32.81
0.7	32.83	32.83	34.41
0.8	34.59	34.00	35.34
0.9	35.35	35.31	36.03
1.0	36.13	36.12	36.62

Table 2: Comparison of the proposed algorithm results with DWT / BPC, DWBT / BPC, *Ridge on EROS*, PSNR(dB)

bpp	$DWT/$ BPC	$DWBT/$ BPC	$DWBT/$ $MBPC$
0.1	30.32	30.77	31.05
0.2	33.49	33.70	34.34
0.3	35.68	35.78	36.75
0.4	37.62	37.51	38.18
0.5	39.13	39.08	39.39
0.6	40.24	40.00	40.98
0.7	41.22	41.15	42.12
0.8	42.20	42.21	42.54
0.9	43.28	43.36	42.54
1.0	44.15	44.35	42.54

sion algorithms are compared in Table 3 and Table 4. The improvement in PSNR of second generation

bandelets over wavelets is evident from the results tabulated. The DWBT with BPC produces PSNR that is higher by 0.1 to 0.5 dB than DWT with BPC. The improvement produced depends on the bit-rate and the test image. At some of the bit-rates, the DWBT / BPC algorithm produces smaller PSNR than the DWT / BPC algorithm. But the bandelets are superior in acquiring the texture information in images, as shown in the visual reconstructions tabulated. The results of progressive compression with the modified algorithm validates the superiority of bandelets over wavelets. The reconstructed images for the above algorithm also demonstrate the efficiency of bandelets in acquiring the texture information in the image as compared to wavelets. The modified algorithm with DWBT increases the PSNR of the reconstructed images in the range of 0.5 to 1.5 dB. This is validated by the quality of the reconstructions. The CR attained by the three algorithms for the two test images are given in Table 5. The results tabulated below show the higher compression capability of the proposed algorithm with bandelets. For both the test images, CR is highest for the proposed algorithm compared to DWT or DWBT with BPC. The results validate the suitability of bandelets for compressing images with texture / geometry.

Table 3: Comparison of reconstructed images with DWT / BPC, DWBT / BPC and DWBT / MBPC, bior4.4 wavelet, *Barbara*, PSNR(dB)

bpp	$DWT/$ BPC	$DWBT/$ BPC	$DWBT/$ $MBPC$
0.1			
0.2			
0.3			
0.4			
0.5			

4 Comparison of Proposed Algorithm with SPIHT

The SPIHT algorithm has been applied on the wavelet-bandelet coefficients to obtain a progressive reconstruction of the input image and it has been compared with the reconstruction using wavelets. Since the bandelet transform is applied on every subband of the wavelet decomposition, the tree structure of wavelets is preserved in the second generation bandelets also. So the SPIHT algorithm will be suitable for compressing the coefficients produced. The geometry in the image will be reflected in all the subbands at the same spatial location. The results of applying SPIHT on DWT coefficients has been compared with DWBT in Table 6 and Table 7 and the corresponding reconstructed images are presented in Table 8 and Table 9.

The PSNR values of the reconstructed images with DWT and DWBT compressed with SPIHT indicate that DWBT is marginally better than DWT, especially for low bit-rates. The quality of the reconstructed images is also superior in case of DWBT than DWT with SPIHT

Table 4: Comparison of reconstructed images with DWT / BPC, DWBT / BPC and DWBT / MBPC, bior4.4 wavelet, *Ridge on EROS*, PSNR(dB)

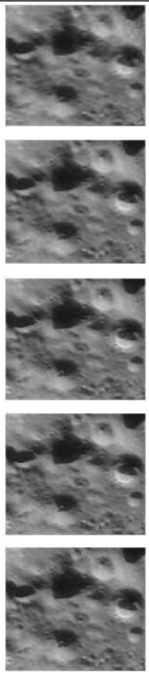
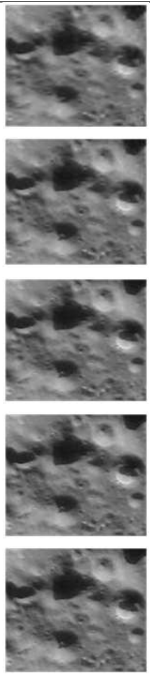
bpp	$DWT/$ BPC	$DWBT/$ BPC	$DWBT/$ $MBPC$
0.1 0.2 0.3 0.4 0.5			

Table 5: Comparison of CR for DWT / BPC, DWBT / BPC, DWBT / MBPC

$Image$	$DWT/$ BPC	$DWBT/$ BPC	$DWBT/$ $MBPC$
$Barbara$	4.23	4.13	4.94
$Ridge$ on $EROS$	6.93	6.67	8.96

Table 6: Comparison of DWT / SPIHT with DWBT / SPIHT and DWBT / MBPC, *Barbara*, PSNR(dB)

bpp	$DWT/$ $SPIHT$	$DWBT/$ $SPIHT$	$DWBT/$ $MBPC$
0.1	20.59	20.72	24.33
0.2	24.02	24.06	26.46
0.3	26.42	26.71	28.53
0.4	28.30	28.47	30.19
0.5	30.67	30.45	31.44
0.6	32.14	31.99	32.81
0.7	33.38	33.16	34.41
0.8	34.67	34.40	35.34
0.9	35.72	35.61	36.03
1.0	36.63	36.55	36.62

algorithm. At higher bit-rates, DWT gives a higher PSNR than DWBT. Comparing SPIHT and the modified bit plane coding with DWBT it is found that for most of the images, the modified algorithm is superior to SPIHT. Therefore, for a given bit-rate and PSNR, modified BPC produces higher compression than SPIHT. The visual quality of the reproduced images also correlate with the PSNR values obtained.

Table 7: Comparison of DWT / SPIHT with DWBT / SPIHT and DWBT / MBPC, *Ridge on EROS*, PSNR(dB)

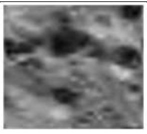
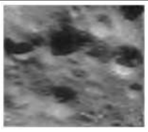
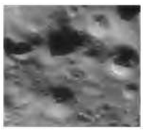
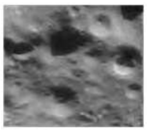
bpp	$DWT/$ $SPIHT$	$DWBT/$ $SPIHT$	$DWBT/$ $MBPC$
0.1	24.10	26.48	31.05
0.2	31.06	31.92	34.34
0.3	34.33	34.76	36.75
0.4	36.44	36.74	38.18
0.5	38.17	38.38	39.39
0.6	39.40	39.59	40.98
0.7	40.45	40.73	42.12
0.8	41.48	41.78	42.54
0.9	42.29	42.55	42.54
1.0	43.07	43.34	42.54

Table 8: Reconstructed Images for DWT / SPIHT with DWBT / SPIHT and DWBT / MBPC, bior4.4 wavelet, *Barbara*, PSNR(dB)

bpp	$DWT/$ $SPIHT$	$DWBT/$ $SPIHT$	$DWBT/$ $MBPC$
0.1 0.2 0.3 0.4 0.5			

The quadtree decomposition and the direction information have to be encoded separately and transmitted along with the compressed DWBT coefficients. This incurs an additional overhead in bits that is not required in wavelets. This is one of the disadvantages of the bandelet transform but it is more than compensated by the superior visual quality of images with textures. Each split in the quadtree requires one bit and the directions

Table 9: Reconstructed Images for DWT / SPIHT with DWBT / SPIHT and DWBT/MBPC, bior4.4 wavelet, *Ridge on EROS*, PSNR(dB)

bpp	*DWT/ SPIHT*	*DWBT/ SPIHT*	*DWBT/ MBPC*
0.1			
0.2			
0.3			
0.4			
0.5			

have to be encoded in a lossless manner. Since the geometry varies from image to image, the number of overhead bits are also variable. The bits required for coding quadtree decomposition and direction information is only a fraction of that required for coding the quantized DWBT coefficients comparatively.

5 Conclusion

The results of applying DWT and DWBT compressed with BPC, SPIHT and the proposed MBPC algorithm on two test images have been compared. It is found that the second generation DWBT acquires the texture information in the images better than DWT. The DWBT coefficients are quantized and encoded using the proposed MBPC and the results are found to be superior to the bit plane coding algorithm as well as SPIHT. The PSNR values are higher for DWBT-MBPC as also the visual quality of reconstruction compared to DWT-BPC, DWBT-BPC and DWBT-SPIHT. The quadtree decomposition and the direction information have to be encoded in a lossless manner and transmitted along with the compressed coefficients. This additional overhead in bits is not required in wavelets and it is one of the disadvantages of the bandelet transform. It is compensated by the superior visual quality of images with textures / geometry. Wavelets introduce visible ringing effects whereas they are concealed in bandelets.

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