A Queueing Network Model for Performance Analysis of Single-Radio and Dual-Radio 802.11 Wireless Mesh Networks

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Abstract. Wireless mesh networks address the growing requirements for networks that are highly scalable and cost-effective, offering end-users large access areas beyond traditional WLAN boundaries, and provide a viable alternative when wired backhaul cannot be supported or afforded. The success of these networks depends on the availability of accurate models and tools for assessing their performances at each layer. However, to date, most analysis of wireless networks has been focused on single-hop and ad hoc multi-hop networks under saturated conditions. In this paper, we introduce an analytical model specifically designed to the performance evaluation of 802.11-based wireless mesh networks under finite load. This model relies on Markov chains for evaluating packet service delays of mesh clients and mesh routers that operate with 802.11 DCF, and open G/G/1 queueing networks for deriving explicit expressions of average end-to-end delay on single-radio and dual-radio WMNs. Our analytical results are verified through extensive simulations. The results quantify the impact of the dedicated backhaul channel and the strong dependency between end-to-end delay and radio range.

Keywords: WMN, 802.11 DCF, end-to-end delay, Markov model, queueing network, diffusion approximation.

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1 Introduction

The popularity of 802.11 technologies has generated a lot of interest in developing wireless networks that support seamless and ubiquitous access across very large areas. Various access points with large coverage are available to these scenarios but their interconnection remains mostly dominated by the deployment of costly wired backhaul networks. Wireless mesh networks (WMN) [1] offer a promising alternative for building robust and reliable wide-area wireless broadband services, in particular when reducing up-front investments is a key concern as in developing countries. Despite the availability of several wireless products and the simplicity of deployment, huge efforts still needed to address the issues behind WMNs. Many WMNs have inherited the MAC and the physical layers of conventional WLANs which have been proved to lack scalability when applied to large areas and multi-hop network settings [1]. Throughput drops significantly as the number of nodes or hops in a WMN increases. Similar problems exist in other networking layers. Consequently, many existing protocols need to be fully assessed in this context to identify the key issues and to guide their further enhancements. In particular, a predictable and low end-to-end delay of data packets is a critical performance issue in many applications expected to be a driving force in the deployment of WMNs: voice over IP, video streaming, videoconferencing, interactive gaming, etc.

The efficiency of the IEEE 802.11 protocol has been subjected to numerous investigations which targeted the modeling of single-hop 802.11 WLANs to find the maximum achievable throughput and to characterize capacity-delay tradeoffs [3, 16, 11]. In his famous work, Bianchi [3] derived a model that incorporates the inherent exponential backoff process as a bidimensional Markov chain and determines the maximum achievable throughput of 802.11 DCF by assuming that every node is saturated and the packet collision probability is constant regardless of the state or station considered. Despite the remarkable accuracy of the obtained results, the saturation assumption is unlikely to be valid in most real networks. Several other papers in the literature have analyzed the performance of multihop 802.11 networks. In [13], Ray and al. quantified the impact of hidden nodes on the performance of linear wireless networks taking into consideration the effects of queueing and retransmissions at each node. Many other studies apply queueing theory for performance analysis of 802.11 networks. In [8], Dong and al. employ a Markov chain model to analyze the probability of transmission in an arbitrary slot at each node in multihop wireless network. The average end-to-end delay is evaluated through the derivation of channel access delay and the queueing delay at each node modeled as an M/G/1 queue.

In this paper, we introduce a detailed analytical model to evaluate the average end-to-end delay in 802.11 WMNs where nodes operate in DCF mode. We study a particular form of WMNs where mesh routers are organized into honey-grid topology providing access to randomly distributed mesh clients which generate finite load traffic. Moreover, our analysis includes two types of WMNs: single-radio mesh and dual-radio mesh. Unlike some existing work, our analysis of end-to-end delay of data packets relies on an accurate modeling of backoff and collision avoidance mechanisms of 802.11 DCF and takes into consideration several prominent parameters: number of radio channels, the offered and the forwarded load across the network, the expected number of hops between source and destination, and the number of interfering nodes whether they are visible or hidden from the point of view of each intermediate node.

The remaining of this paper is outlined as follows. In section 2, we introduce our network model topology, provide some background about the 802.11 DCF mechanism and introduce the Markov model employed for the evaluation of channel access and transmission delays. In section 3, we give some insights about the well known diffusion approximation and describe the proposed open queueing network for end-to-end delay analysis. Section 4 evaluates the accuracy of our model by comparing the analytical results with those obtained by mean of simulation. The discussion of the various results leads to some considerations about the influential factors that determine end-to-end delay in single-radio and dual-radio WMNs. Concluding remarks and some perspectives are given in section 5.

2 Delay Analysis of 802.11 DCF

As depicted in figure 1, The mesh routers are distributed over a two-dimensional area with limited size called the service area. Their positions constitute a regular lattice called the honey-grid model that enables the radio coverage of all mesh clients disseminated throughout the service area. As stated in [9], this model minimizes the number of expected hops between mesh routers, and enables a perfect overlapping between the positions of the routers on the lattice and the positions of their interfering nodes.The size of the network can be expressed in terms of k co-centered hexagonal rings around the central node 0, or by N the total number of nodes in this configuration. In addition, m mesh clients are uniformly and independently distributed inside the service area delimited by the outmost ring. The radio range of each client is defined by αR, where R is the maximum achievable radio range and $\alpha \in [0, 1]$. We define the value of α in such a way that mesh clients are always covered by at least one mesh router and at most 3 mesh routers.

2.1 Single-Radio Mesh

In this scenario, mesh routers and mesh clients share the same 802.11 radio channel. Thus, the co-located client/router and router/router transmissions interfere with each other. We assume that each mesh client generates traffic of fixed-size data packets following a Poisson processus of average rate λ_c, destined to a randomly chosen mesh client. The wireless channel is considered as error-free.

The CSMA/CA function of the 802.11 DCF based on binary exponential backoff is modeled through the Markov chain depicted in figure 2. This chain was first proposed in the original work of Bianchi [3], and then extended by [8] to deal with non-saturated traffic conditions. It may be described as follows: the state *Empty* occurs when the node has a empty queue of packets waiting for transmission, whereas the state *FirstPkt* denotes a node which receives its first packet for transmission. The states (i, j) represent a node in the backoff mode, where i denotes the backoff stage, and j denotes the value of the backoff window which is uniformly chosen in the range $(0, W_i)$. The backoff window size W_i is equal to $2^i W_0$, where W_0 is the initial window

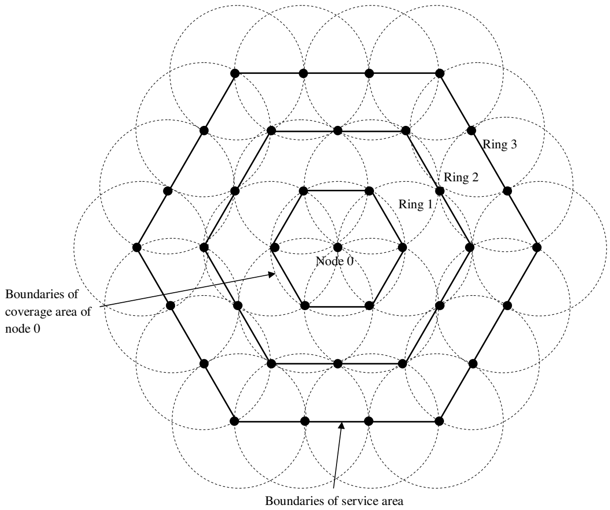

Figure 1: Honey grid model .

size. As we assume that the number of retransmission in infinite, this chain has a infinite number of states (i, j).

The states $(0', j)$ on the left of the figure denote the post-backoff procedure which runs after a successful transmission. The 802.11 DCF standard [10] specifies that the node must executes in this case a backoff procedure with the initial windows value W_0 even if there is no packets waiting in the queue. If no packet arrives during this time, which may happens with a probability P_{nk}, the node returns to the state *Empty* and senses the channel, otherwise it continues with the regular backoff procedure. We use one instance of this Markov chain for modeling the channel access of mesh clients and another one for modeling the channel access of mesh routers because the expected handled traffic and the number of interfering nodes are not equal in these two cases.

Let define the probabilities associated with the Markov chain's transitions of mesh client. P_{idle1} denotes the probability that no transmission occurs in the radio range of a mesh client in a randomly chosen system time slot of duration σ, it is given by:

$$P_{idle1} = (1 - \tau_c)^{n_{cc}+1} \cdot (1 - \tau_r)^{n_{cr}} \quad (1)$$

where τ_c and τ_r are the probabilities of a transmission in the time slot of a mesh client and a mesh router, respectively. n_{cr} is the expected number of neighbor mesh routers within the coverage area of a mesh client, and n_{cc} is the expected number of mesh clients within the radio range of a mesh client. P_{col1} denotes the probability that a transmitted frame encounters a collision, it is given by:

$$P_{col1} = 1 - (1 - \tau_c)^{n_{cc}+h_{cc}} \cdot (1 - \tau_r)^{n_{cr}+h_{cr}} \quad (2)$$

where h_{cc} is the expected number of hidden mesh clients from a mesh client, and h_{cr} is the expected number of hidden mesh routers from a mesh client. A hidden node from a given node is a neighbor of any neighbor of this node without being its direct neighbor. Note we have derived explicit formulas for n_{cc},n_{cr}, h_{cc} and h_{cr} but we do not given them here for the sake of conciseness.

P_{e1} denotes the probability that a mesh client has an empty queue. Considering an Poisson arrival process and an average packet service time, $E[C]$, following a general distribution, P_{e1} is given by:

$$P_{e1} = 1 - \lambda_c \cdot E[C] \quad (3)$$

with $\lambda_c \cdot E[C] \leq 1$. From the balance equations of the chain in the steady state, we obtain the following relationships:

$$b_{i,0} = P_{col1}^{i} \cdot b_{0,0} \quad (4)$$

$$b_{\acute{0},k} = \frac{W_0 - k}{W_0} \cdot P_{e1} \cdot b_{succ} \quad (5)$$

$$b_{i,k} = \frac{W_i - k}{W_i} \cdot P_{col1}^{i} \cdot b_{0,0} \quad (6)$$

$$\tau_c = b_{0,0}\left(\frac{1}{1 - P_{col1}}\right) + b_{empty} \cdot P_{idle1}\left(1 - e^{-\lambda_c \sigma}\right) \quad (7)$$

The probability b_{empty} that the current state is the state *Empty* is given by:

$$b_{empty} = \frac{b_{0,0} \cdot P_{e1} \cdot P_{nk1}}{1 - P_{idle1} \cdot P_{e1} \cdot P_{nk1} \cdot (1 - P_{col1})} \quad (8)$$

The probability P_{nk1} may be expressed as

$$P_{nk1} = exp\left(-\frac{\lambda_c \cdot (W_0 + 1) \cdot \bar{\sigma_1}}{2}\right) \quad (9)$$

where $\bar{\sigma_1}$ denotes the average time between successive timer decrements when the node is the backoff mode. This timer is frozen each time the node detects an activity in the channel [10]. Substituting (8) in (7), we get:

$$\tau_c = b_{0,0} \cdot \left[\frac{1}{1 - P_{col1}} + \frac{P_{e1} \cdot P_{nk1} \cdot P_{idle1} \cdot \left(1 - e^{-\lambda_c \sigma}\right)}{1 - P_{idle1} \cdot P_{e1} \cdot P_{nk1} \cdot (1 - P_{col1})}\right] \quad (10)$$

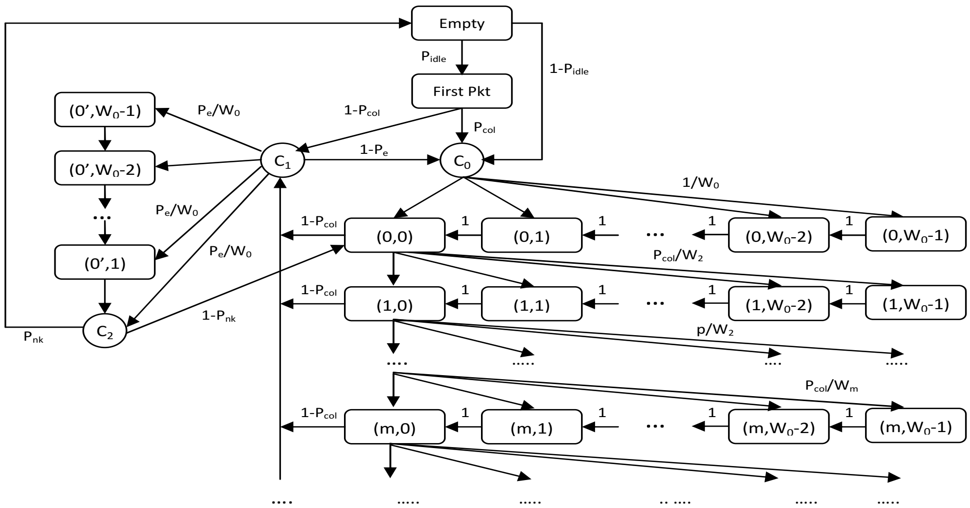

Figure 2: Finite Load Markov Chain of 802.11 DCF .

Applying the normalization condition of the chain, we obtain:

$$\sum_{i=0}^{\infty}\sum_{k=0}^{W_i-1} b_{i,k} + \sum_{k=0}^{W_0-1} b_{\acute{0},k} + b_{empty}\cdot(1+P_{idle1}) = 1$$

By substituting (5), (6) and (8) into this equation:

$$\frac{b_{0,0}}{2}\cdot\left[\frac{W_0+1-(W_0+2)\cdot P_{col1}}{(1-P_{col1})\cdot(1-2P_{col1})}+ \frac{P_{e1}\cdot(2P_{nk1}\cdot(P_{idle1}+1)+W_0-1)}{1-P_{idle1}\cdot P_{e1}\cdot P_{nk1}\cdot(1-P_{col1})}\right]=1 \quad (11)$$

Let P_{tr1} denotes the probability that at least one node transmits within the radio range of the mesh client when this latter is in the backoff mode. It can be expressed as

$$P_{tr1} = 1-(1-\tau_c)^{n_{cc}}\cdot(1-\tau_r)^{n_{cr}} \quad (12)$$

The probability P_{succ1} that this transmission succeeds is

$$P_{succ1} = \Big[n_{cc}\cdot\tau_c\cdot(1-\tau_c)^{n_{cc}-1}\cdot(1-\tau_r)^{n_{cr}} + n_{cr}\cdot\tau_r\cdot(1-\tau_r)^{n_{cr}-1}\cdot(1-\tau_c)^{n_{cc}}\Big]\times \frac{1}{1-(1-\tau_c)^{n_{cc}}\cdot(1-\tau_r)^{n_{cr}}} \quad (13)$$

Thus, the parameter $\bar{\sigma_1}$ can be expressed now as follows:

$$\bar{\sigma_1} = (1-P_{tr1})\cdot\sigma + P_{tr1}\cdot P_{succ1}\cdot(T_s+\sigma) + P_{tr1}\cdot(1-P_{succ1})\cdot(T_c+\sigma) \quad (14)$$

where T_s is the length of a successful transmission time slot, and T_c is the length of a collision time slot. Let $Size(P)$ be the size of data packets, H_{phy} and H_{mac} be the transmission durations of physical and MAC packet headers, respectively, and δ be the propagation delay, T_s and T_c are given by:

$$T_s = DIFS + RTS + CTS + H_{mac} + Size(P) + ACK + 3SIFS + 4H_{phy} + 4\delta \quad (15)$$

$$T_c = RTS + CTS + SIFS + DIFS + 2H_{phy} + 2\delta \quad (16)$$

As we assume that all mesh routers are equal, the probability that a traffic flow transits by a randomly chosen router is $\frac{e_h+1}{N}$, where e_h is the expected number of router to router hops traversed by data packets. Hence, the expected average arrival traffic on a mesh router is given by:

$$\lambda_r = \lambda_c\cdot M\cdot\frac{e_h+1}{N} \quad (17)$$

By symmetry with the results obtained for mesh clients, we derive in a straightforward manner a set of corresponding relations for mesh routers.

Expression of packet service time In 802.11 DCF, the packets arriving at a node experience a different medium access delay depending on whether or not the queue is empty. If the queue in non empty, the node enters a backoff procedure, as illustrated by figure 2, before trying to transmit the packet. Otherwise, one of the following three events can occur: (i) the medium is idle and the node transmits the packet successfully; (ii) the medium is idle but a collision occurs at transmission, the node returns to backoff mode before it retransmits; (iii) the medium is busy, the node goes to the backoff mode before it transmits. Therefore, considering that the node is a mesh client, the service time of a packet arriving to an empty queue, $E[C_e]$, may be expressed as:

$$E[C_e] = (1 - P_{idle1}) \cdot E[C_e] + P_{idle1} \times \left(P_{col1} \cdot (T_c + E[C_{ne}]) + (1 - P_{col1}) \cdot T_s \right) \quad (18)$$

where $E[C_{ne}]$ is the service time of a packet arriving to a non empty queue. This latter is the sum of the total durations needed to reach a transmission state at each attempt, the total collision time and the transmission time. Let β represents the number of retransmissions, the probability that the transmission succeeds after r retransmissions is given by:

$$P(\beta = r) = P_{col1}^r \cdot (1 - P_{col1})$$

From the Markov chain, the conditional probability of $E[C_{ne}]$ given r retransmission can be expressed as

$$\begin{aligned} E[C_{ne} \mid \beta = r] =& T_s + \bar{\sigma_1} \sum_{i=0}^{r} \sum_{j=0}^{W_i - 1} \frac{1}{W_i} j + r \cdot T_c \\ =& T_s + \frac{\bar{\sigma_1}}{2} \left(W_0 \cdot \left(2^{r+1} - 1 \right) - r - 1 \right) \\ & + r \cdot T_c \end{aligned}$$

Hence,

$$E[C_{ne}] = \sum_{i=0}^{\infty} E[C_{ne} \mid \beta = i] \cdot Pr(\beta = i)$$

We find that $E[C_{ne}]$ is convergent only when $P_{col1} < \frac{1}{2}$ to:

$$\begin{aligned} E[C_{ne}] =& T_s + T_c \cdot \frac{P_{col1}}{1 - P_{col1}} + \\ & \bar{\sigma_1} \frac{W_0 \cdot (1 - P_{col1}) - 1 + 2P_{col1}}{2\,(1 - 2P_{col1}) \cdot (1 - P_{col1})} \end{aligned} \quad (19)$$

The average service time $E[C]$ depends in the general case on the state of the node's queue:

$$\begin{aligned} E[C] &= (1 - P_{e1}) \cdot E[C_{ne}] + P_{e1} \cdot E[C_e] \\ &= ((P_{col1} - 1) \cdot P_{e1} \cdot P_{idle1} + 1) \cdot E[C_{ne}] + \\ & \quad P_{e1} \cdot P_{idle1} \cdot (P_{col1} \cdot T_c + (1 - P_{col1}) \cdot T_s) \end{aligned} \quad (20)$$

We can derive in a similar way $E[R]$, the average service time on a mesh router. The previously defined variables related to the mesh clients P_{idle1}, P_{col1}, P_{e1}, P_{nk1}, τ_c, σ_1, $E[C]$, and the corresponding ones related to the mesh routers, along with the normalization equations of the chains, can be combined to form a non-linear system the following constraints: $\lambda_c \cdot E[C] \leq 1$, $\lambda_r \cdot E[R] \leq 1$, $P_{col1} < \frac{1}{2}$ and $P_{col2} < \frac{1}{2}$, where P_{col2} is the probability of collision when the frame is transmitted by a mesh router. This system can be solved using numerical techniques.

The second moment of service time An accurate analysis of the average packet delay in a queueing network requires the knowledge of the second moment of packet service time. First, let define the second moment of service time of a packet arriving at an empty queue of a mesh client:

$$\begin{aligned} E[C_e^2] =& (1 - P_{idle1}) \cdot E[C_{ne}^2] + P_{idle1} \cdot (1 - P_{col1}) \\ & \cdot T_s^2 + P_{idle1} \cdot P_{col1} \cdot E[(C_{ne} + T_s)^2] \end{aligned}$$

The second moment of service time of a packet arriving at a non empty queue can be obtained as follows:

$$\begin{aligned} E[C_{ne}^2] =& \sum_{j=0}^{\infty} E[C_{ne}^2 \mid \beta = j] \cdot Pr(\beta = j) \\ =& \sum_{j=0}^{\infty} \sum_{i=0}^{j} VAR[B_i] \cdot Pr(\beta = j) + \\ & \sum_{j=0}^{\infty} \left(\sum_{i=0}^{j} E[B_i] + j \cdot T_c + T_s \right)^2 Pr(\beta = j) \end{aligned} \quad (21)$$

where B_i is the duration of the backoff period at the i^{th} retransmission attempt. It is given by:

$$B_i = \sum_{j=0}^{w_i} \sigma_i \quad (22)$$

where σ_i is the duration of the backoff slot when the backoff window decrements from $i+1$ to i, and w_i is the size of the backoff window uniformly selected between 0 and $2^i W_0 - 1$. $E[B_i]$ and $VAR[B_i]$ are obtained as follows:

$$E[B_i] = \bar{\sigma_1} \cdot E[w_i] = \bar{\sigma_1} \cdot \frac{2^i W_0 - 1}{2} \quad (23)$$

where $\bar{\sigma_1}$ is given in (14). Also, we have

$$\begin{aligned} VAR[B_i] =& E[B_i^2] - E[B_i]^2 \\ =& E\left[\left(\sum_{j=0}^{w_i} \sigma_i\right)^2\right] - \bar{\sigma}_1^2 \cdot \frac{\left(2^i W_0 - 1\right)^2}{4} \\ =& VAR[\sigma_i] \cdot E[w_i] + \bar{\sigma}_1^2 \cdot E[w_i^2] - \\ & \bar{\sigma}_1^2 \cdot \frac{\left(2^i W_0 - 1\right)^2}{4} \end{aligned}$$

As w_i is uniformly distributed, it is easy to get:

$$E[w_i^2] = \frac{2^i W_0 \cdot (2^{i+1} W_0 - 1)}{6} \tag{24}$$

The variance of σ_i is:

$$VAR[\sigma_i] = \bar{\sigma_2} - \bar{\sigma}_1^2 \tag{25}$$

with

$$\begin{aligned} \bar{\sigma_2} =& (1 - P_{tr1}) \cdot \sigma^2 + P_{tr1} \cdot P_{succ1} \cdot (T_s + \sigma)^2 \\ &+ P_{tr1} \cdot (1 - P_{succ1}) \cdot (T_c + \sigma)^2 \end{aligned} \tag{26}$$

Thus, $VAR[B_i]$ can be expressed as follows:

$$\begin{aligned} VAR[B_i] =& \bar{\sigma}_1^2 \cdot \frac{(2^i W_0 - 1) \cdot (2^i W_0 - 5)}{12} \\ &+ \bar{\sigma_2} \frac{2^i W_0 - 1}{2} \end{aligned} \tag{27}$$

By substituting (23) and (27) in (21), and after some straightforward algebra, we find a lengthy expression of $E[C_{ne}^2]$ which converges when $P_{col1} < \frac{1}{4}$.

The second moment of packet service time in the general case can be expressed as follows:

$$E[C^2] = P_{e1} \cdot E[C_e^2] + (1 - P_{e1}) \cdot E[C_{ne}^2] \tag{28}$$

The second moment of packet service time at a mesh router can also be obtained in a similar way.

2.2 Dual-Radio Mesh

In this second scenario, the mesh routers have two radios operating on different frequencies. One radio is used for mesh client access and the other radio provides wireless backhaul between routers. The radios operate in orthogonal frequency channels so they can run concurrently without interference. The two radio channels are occupied in the following way:

(i) *Client channel*: the mesh clients contend from one part with other neighbor mesh clients transmissions and from the other part with the transmissions from neighbor mesh routers to mesh clients. The probability that a randomly chosen mesh router is the first router on the path for a given mesh client's traffic is $\frac{1}{N}$. Hence the average arrival rate λ_{rc} at the router from its client radio interface is equal to $\lambda_c \cdot \frac{M}{N}$.

(ii) *Backhaul channel*: the mesh routers contend on this channel to forward client packets. The average arrival rate λ_{rr} is equal to $\lambda_c \frac{e_h \cdot M}{N}$ considering that the generated traffic of a given client is forwarded on average e_h times inside the WMN.

Average packet service times of mesh clients and mesh routers using client channel, $E[C]$ and $E[R_c]$, respectively, can be determined by substituting λ_r with λ_{rc} in the non-linear system defined previously in the case of single-radio mesh. Whereas, the average service time of mesh routers on backhaul channel, $E[R_r]$, depends only on the contention with interfering mesh routers.

3 End-to-End Delay in WMNs

3.1 End-to-End Delay Analysis

Packet end-to-end delay in a WMN can be evaluated by the sum of sojourn times at the source mesh client and intermediate mesh routers. We derive in this section closed-form expressions of packet end-to-end delay in single-radio and dual-radio configurations.

Single-radio mesh We model the single radio WMN as a queueing network with two types of stations: M client stations, numbered from 1 to M, which act as the entry point of external jobs corresponding to data packets, and N router stations, numbered from $M+1$ to $M+N$, which either forward data packets to other router stations or either absorb them if they are the last router towards the packet destination. Note that once a packet leaves a client station after its service completion, it goes to a router station and doesn't return again to a client station.

Let assume that a given traffic with an effective rate of λ_c enters to each client station constituting a total external traffic with average rate of $M \cdot \lambda_c$. Hence, the visit ratio of a client station i could be given by $e_i = \frac{1}{M}$. All routers stations in the network have an equal chance to be an intermediate station of a given packet, hence considering that packets transits e_h times on average between router stations before reaching the destination, the visit ratio of the router station i can be expressed as $e_i = \frac{e_h+1}{N}$. Considering that all router stations are similar, the forwarding probability p_{ij}, corresponding to the probability that a packet is transferred from client station i to router station j, is given by $p_{ij} = \frac{1}{N}$. By the same similarity rule, we could express the probability that a mesh router absorbs a packet as $\frac{1}{N}$. Thus, the forwarding probability, denoted by q_{ij}, corresponding to the probability that a packet is transferred between the router stations i and j ($i \neq j$) is the joint probability that i doesn't absorb the packet and also transfer it to j. It is given by:

$$q_{ij} = \left(1 - \frac{1}{N}\right) \cdot \frac{1}{N-1} = \frac{1}{N} \tag{29}$$

Using the diffusion approximation [5], we evaluate the squared coefficient of variation of interarrival times at router station i as

$$\begin{aligned} c_{Ai}^2 =& 1 + \sum_{j=1}^{M} \left(c_{Bj}^2 - 1\right) \cdot p_{ji}^2 \cdot e_j \cdot e_i^{-1} \\ &+ \sum_{j=M+1, j\neq i}^{M+N} \left(c_{Bj}^2 - 1\right) \cdot q_{ji}^2 \cdot e_j \cdot e_i^{-1} \end{aligned} \tag{30}$$

Let c_{Bc}^2 and c_{Br}^2 denote the squared coefficient of variation of service times of client stations and router stations, respectively. Assuming that all routers are identical, the squared coefficient of variation of interarrival times at a router station is hence given by

$$c_{Ar}^2 = 1 + \frac{c_{Bc}^2 - 1}{N \cdot (e_h + 1)} + (N-1) \cdot \frac{c_{Br}^2 - 1}{N^2} \tag{31}$$

where from the results obtained in section 2.1, c_{Bc}^2 and c_{Br}^2 are given by:

$$c_{Bc}^2 = \frac{E[C^2] - E[C]^2}{E[C]^2} \tag{32}$$

$$c_{Br}^2 = \frac{E[R^2] - E[R]^2}{E[R]^2} \tag{33}$$

Also according to the diffusion approximation, the following approximated expression for the steady state probabilities of the number of jobs at a router station can be obtained:

$$\hat{\pi}(k) = \begin{cases} 1 - \rho_r & k = 0 \\ \rho_r \cdot (1 - \hat{\rho}_r) \cdot \hat{\rho}_r^{\,k-1} & k > 0 \end{cases} \tag{34}$$

with :

$$\rho_r = \frac{M \cdot \lambda_c \cdot (e_h + 1) \cdot E[R]}{N} \tag{35}$$

$$\hat{\rho}_r = exp\left(-\frac{2(1-\rho_r)}{c_{Ar}^2 \cdot \rho_r + c_{Br}^2}\right) \tag{36}$$

From little law's [2], the average sojourn time of packets at mesh routers, denoted $\bar{T}_r$, can now be derived as

$$\bar{T}_r = \frac{E[R]}{1 - \hat{\rho}_r} \tag{37}$$

On the other hand, as the arrival traffic at client stations follows an exponential distribution, we can use an M/G/1 queue to analyze their average service delay. According to the Pollaczek-Khinchin formula [2], the expected waiting delay, denoted $\bar{W}_c$, at client stations is

$$\bar{W}_c = \frac{\lambda_c \cdot E[C^2]}{1 - \lambda_c \cdot E[C]} \tag{38}$$

Therefore, the average end-to-end packet delay, $E[D]$, in a single-radio mesh is the sum of queuing and packet service times at the source client station, and sojourn times at intermediate router stations:

$$E[D] = \bar{W}_c + E[C] + (e_h + 1) \cdot \bar{T}_r \tag{39}$$

Dual-radio mesh The modeling of the dual-radio configuration is accomplished by inserting N other stations, numbered from $M+N+1$ to $M+2N$, to the previous queueing network. These stations, referred to as last-hop stations, represent mesh routers transmitting packets to a mesh client using the client channel. Packets arrive to these stations either from client stations or either from backhaul stations which represent mesh router transmitting on backhaul channel. Last-hop stations absorb all the received packets. We refer to the router stations in the network by backhaul stations.

Data packets go through one hop less on average in backhaul channel than in single-radio configuration. Hence, the visit ratio of a backhaul station i becomes in this case $e_i = \frac{e_h}{N}$. Likewise, the visit ratio of a last-hop station i corresponds to the probability that a given router is the last one towards the recipient client, which is given by $e_i = \frac{1}{N}$.

The forwarding probabilities between the stations of the network can be expressed as follows: from client station to backhaul station $\frac{N-1}{N^2}$, from client station to last-hop station $\frac{1}{N^2}$; from backhaul station to backhaul station $\frac{1}{N}$; and from from backhaul station to last-hop station $\frac{1}{N(N-1)}$.

Therefore, the squared coefficient of variation of interarrival times at backhaul stations and last-hop sta-

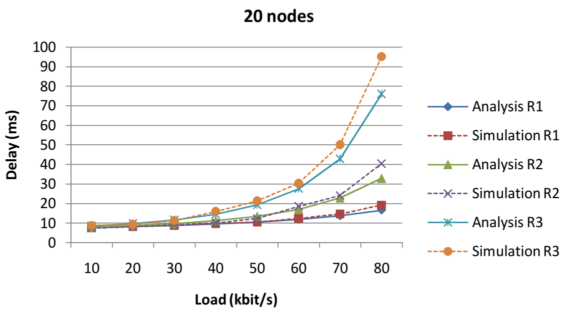

Figure 3: Analy. vs. sim. average delays with 1 ring.

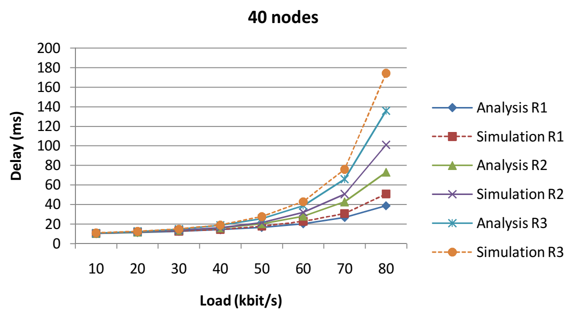

Figure 4: Analy. vs. simu. average delays with 2 rings.

tions, respectively c^2_{Arr} and c^2_{Arc}, may be expressed as:

$$c^2_{Arr} = 1 + \frac{(N-1)^2\left(c^2_{Bc}-1\right)}{N^3 e_h} + \frac{(N-1)\left(c^2_{Brr}-1\right)}{N^2} \quad (40)$$

$$c^2_{Arc} = 1 + \frac{c^2_{Bc}-1}{N^3} + \frac{e_h\left(c^2_{Brr}-1\right)}{N^2(N-1)} \quad (41)$$

where c^2_{Brr} denotes the squared coefficient of variation of service time at backhaul stations. Consequently, the average service delays at backhaul stations and last-hop stations, respectively $\bar{T}_{rr}$ and $\bar{T}_{rc}$, may be derived as:

$$\bar{T}_{rr} = \frac{E[R_r]}{1-\hat{\rho}_{rr}} \quad (42)$$

$$\bar{T}_{rc} = \frac{E[R_c]}{1-\hat{\rho}_{rc}} \quad (43)$$

where $E[R_r]$ and $E[R_c]$ denote the average packet service time at backhaul stations and last-hop stations, respectively, and

$$\hat{\rho}_{rr} = exp\left(-\frac{2(1-\rho_{rr})}{c^2_{Arr}\cdot\rho_{rr} + c^2_{Brr}}\right) \quad (44)$$

$$\hat{\rho}_{rc} = exp\left(-\frac{2(1-\rho_{rc})}{c^2_{Arc}\cdot\rho_{rc} + c^2_{Brc}}\right) \quad (45)$$

$$\rho_{rr} = \frac{M\cdot\lambda_c\cdot e_h\cdot E[R_r]}{N} \quad (46)$$

$$\rho_{rc} = \frac{M\cdot\lambda_c\cdot E[R_c]}{N} \quad (47)$$

Finally, the expression of average end-to-end packet delay in a dual-radio wireless mesh is:

$$E[D] = \bar{W}_c + E[C] + e_h\cdot\bar{T}_{rr} + \bar{T}_{rc} \quad (48)$$

Note that $E[C]$ refers here to the average packet service time at client station in a dual-radio configuration.

4 Numerical Results

We present in this section some results of both analysis and simulation to evaluate the effectiveness of the proposed analytical model. The mean and the second moment of channel access delays are computed by solving the non-linear systems defined in section 2 using the IPOPT software package for large-scale nonlinear optimization [15]. Simulations are performed using NS-2 [12]. We define two network topologies: (i) a one-ring network with 7 mesh routers and a number of mesh clients varying from 10 to 20 nodes; (ii) a two-ring network with 19 mesh routers and 40 mesh clients. The mesh clients are randomly distributed into the service area. The radio range of clients varies from $R1 = (\frac{\sqrt{3}}{3}+0.1)R$, $R2 = (\frac{\sqrt{3}}{3}+0.2)R$, to $R3 = (\frac{\sqrt{3}}{3}+0.25)R$ in each scenario. An exponential traffic source is attached to mesh clients producing packets of length 2000 bytes at several rates. Each mesh client selects a random mesh client destination. As a mean to exclude the mesh clients from participating in packet forwarding, the routes are built and maintained using the OLSR-UM [14] which implements the base specification of the proactive routing protocol OLSR [7] for NS-2 simulator. OLSR includes a very useful option, called *willingness*, which specifies the willingness of a node to carry and forward traffic for other nodes. Also, we minimize the frequency of exchanged routing updates to reduce their impact on data traffic. We summarize in table 1 the parameters used in both simulations and analytical model corresponding to the DSSS PHY layer with a raw bit rate of 11 Mbit/s.

Figure 3 and 4 plot the results of average end-to-end delays of the analytical model faced to those obtained from NS-2 simulation. We observe in case of light traffic, the analytical model provides accurate results for each WMN and each radio range. However, as the traffic load in the network increases, the average end-to-end delay of simulation becomes significantly larger than the results obtained through the analytical

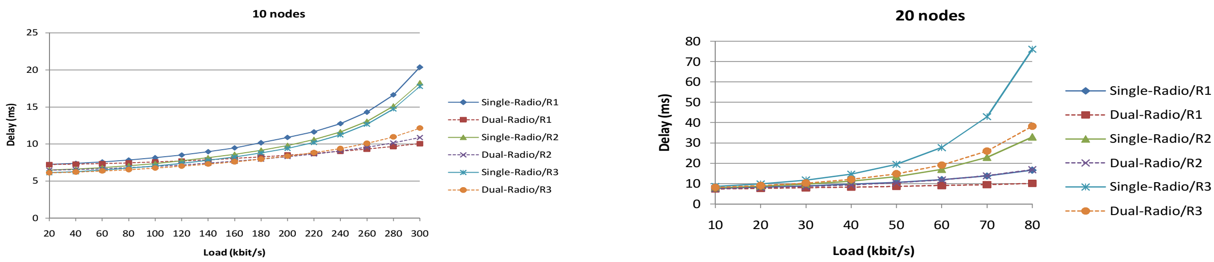

Figure 5: Comparison of average end-to-end delays in 1-ring WMN.

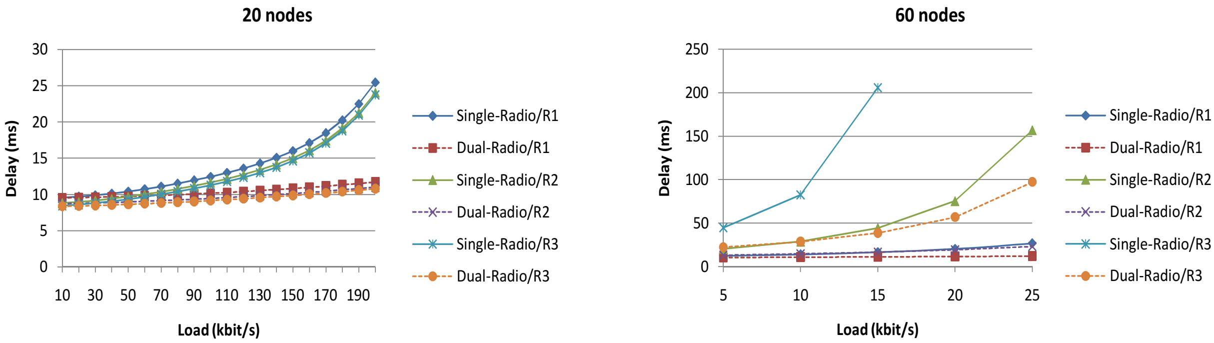

Figure 6: Comparison of average end-to-end delays in 2-ring WMN.

Table 1: PHY and MAC system parameters.

Packet payload	2000 bytes
MAC header	272 bits
PHY header	192 bits
RTS	160 bits + PHY header
CTS	112 bits + PHY header
ACK	112 bits + PHY header
Channel Bit rate	11 Mbps
Preambule rate	1 Mbps
Propagation Delay	1 μs
Slot Time	20 μs
SIFS	10 μs
DIFS	50 μs

model. This divergence has been also outlined in [4]. The main reason is that simulations rely on shortest path routing which may result in heavily loaded mesh routers across the network, whereas the analytical model relies on a probabilistic routing where the traffic is spread uniformly over the mesh routers.

Figures 5 and 6 compare the average end-to-end delays obtained with single-radio and dual-radio WMNs in 1-ring and 2-ring WMNs, respectively. The solid lines are for single-radio delays while the dotted lines are for dual-radio delays. The results show clearly that average delay depends on network load, the density of the network and the expected number of hops. In each configuration, the average delay grows exponentially as the network load approaches the saturation throughput. As demonstrated in [6], these results confirm that the binary exponential backoff tends to be very harmful in saturated multi-hop network where each packet is repeated by the adjacent router.

Theses figures show without surprise that in almost all cases dual-radio settings keep the average delay lower than single-radio do as the network load increases. This can be explained by the fact that as the load increases, the collision probability of packets, and therefore the backoff stage, increases more slowly in dual-radio where the forwarded packets are transmitted on a separate channel. The benefit of having two radio channels becomes even more substantial as the size of the network increases because the forwarded packets traverse more hops on average and consequently preventing mesh clients from accessing backhaul channel contribute to reducing delay more noticeably.

Furthermore, we observe that radio range of mesh clients is another predominant factor. When the network density is low, e.g. 1-ring WMN with 10 nodes, 2-ring WMN with 20 nodes, the highest radio range turns out to be most suitable because the gain from traversing less hops in average overcomes the inconvenient of having more interfering neighbors. This observation in inverted when the network density is high, e.g. 1-ring WMN with 20 nodes, 2-ring WMN with 60 nodes.

Moreover, it's interesting to note that some cases reveal that a lowest radio range have more impact in reducing delay than using a second radio channel. As an example, in a 1-ring WMN with 20 nodes, the dual-radio average delay using range $R3$ is higher than the single-radio one using ranges $R1$ or $R2$.

5 Conclusion

This paper presented an analytical model using Markov chains and queueing networks to express the average end-to-end delay of WMNs operating with 802.11 DCF, the most widely accepted standard for wireless LANs. The derivation of average packet service times considers carefully the effects of visible and hidden interfering nodes. The proposed open G/G/1 networks determine the average waiting times in the queues of mesh clients and mesh routers, hence enabling the evaluation of end-to-end delays. The analytical results, which were validated through extensive simulations, show the impact of network density, the generated load on each mesh client, and the client's radio ranges on the delay. We have pointed out the dramatic effect of using a separate backhaul channel on reducing end-to-end packet delays, particularly in large WMNs with low density of clients. Furthermore, we have exhibited that using high radio range in single-radio mesh is the best option in term of reducing delay when the number of nodes is low, whereas using a low radio range is the best one in the opposite case. The proposed model leads to several venues for future work. Our current directions include end-to-end delay and throughput analysis of multi-radio WMNs and the analysis of other channel access mechanisms, such as 802.11s. We envisage also to integrate more advanced routing protocols and QoS mechanisms in our model and study their effects on performance.

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